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Dynamical Scaling Behaviour Far from Equilibrium

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Ageing phenomena and dynamical scaling behaviour are studied both in ferromagnets and in critical spin glasses. For the ferromagnetic systems we find that dynamical scaling functions measured after a quench into the ordered low temperature phase are in complete agreement with predictions coming from the recently proposed theory of local scale invariance. For a quench to the critical point corrections to these predictions are shown to exist. Critical spin glasses are found to display the same ageing phenomenology as critical ferromagnets. Numerical evidence indicates that the concept of universality is weaker in critical spin glasses than in critical ferromagnets.

1 Introduction

Ageing phenomena are encountered in a large variety of out-of-equilibrium systems (see[1] for a recent review). Well-known examples are found in glasses, spin glasses, polymers and colloids, but also simple ferromagnets, quenched to or below their critical point, display this intriguing behaviour. In praxis ageing behaviour is used in order to change the properties of materials undergoing a quench from high to low temperatures.

Many of the investigations in this field study the dynamical scaling behaviour often encountered in systems being far from equilibrium. Dynamical scaling follows directly from the existence of a unique typical dynamical length scale which increases in time with a power law. The power law behaviour is thereby due to the presence of slow degrees of freedom.

The description of ageing phenomena starts from the observation that dynamical correlation and response functions transform covariantly under the dynamical scale transformation $t \longrightarrow (1+\varepsilon)^z t$, $\vec{r} \longrightarrow (1+\varepsilon)\vec{r}$. Here t is time, $\vec{r}$ denotes the space point, and z is the dynamical exponent. It has been shown recently[2,3] that the dynamical symmetry with ε constant can be generalized to a local symmetry with $\varepsilon = \varepsilon(t, \vec{r})$. Starting from this generalized space-time symmetry exact expressions for dynamical response and correlation functions have been derived for the first time[2,4,5].

In the last years I studied the dynamical scaling behaviour of nonequilibrium systems through extensive numerical simulations. The purpose of this study was two-fold. On the one hand the predictions coming from the theory of local scale invariance were confronted with numerical data obtained for ferromagnets quenched to temperatures equal or less than the critical temperature. This yielded interesting results on the applicability of the concept of generalized space-time symmetries to nonequilibrium systems. On the other hand the investigation of the phenomenology of ageing was extended to other, more complex, systems as for example spin glasses quenched to their critical point.

As discussed in the following, these numerical studies have yielded new insights into the universal features of the dynamical behaviour of systems brought out of equilibrium by a sudden change of external conditions.

2 Ageing Phenomena in Ferromagnets

In order to better understand ageing, consider a ferromagnet prepared at very high temperatures in a completely disordered state and then quenched to low temperatures. In case the final temperature is below the critical temperature, phase ordering sets in, and, due to the competition of different ordered equilibrium states, formation of domains take place. This is shown in Figure 1 for the case of an uniaxial ferromagnet with two competing ordered states. The typical size $L(t)$ of the domains increases as a power law of time t: $L(t) \sim t^{1/z}$. The exponent z, which is called dynamical exponent, takes on the value 2 when no quantities are conserved. The slow degrees of freedom, responsible for the ageing phenomena, are thereby provided by the movement of the walls separating the different domains.

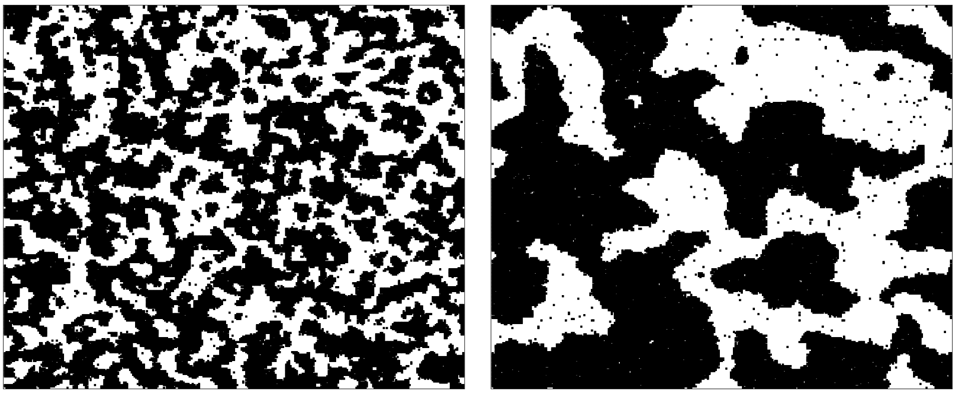

Figure 1. Formation of domains in an uniaxial ferromagnet with two competing ordered states (black and white patches) quenched to temperatures below the critical temperature. The typical size of the domains increases as a function of time (from left to right).

Ageing manifests itself most readily through the behaviour of two-time quantities as dynamical correlation and response functions. Indeed, in the ageing regime these quantities depend in a complicated way on both times and not only on the time difference, as this is the case at equilibrium. To be specific, consider the autocorrelation function $C(t, s)$ which measures to what extent configurations at two times s and $t > s$ are correlated. As shown in Figure 2a for the case of a quench of the two-dimensional Ising model (a very simple model which nevertheless captures most of the physics of uniaxial ferromagnets) to low temperatures the autocorrelation is not a simple function of the time difference $t - s$. In fact, the decay of C is the slowest for the largest value of s. It is this behaviour which we call ageing.

A characteristic behaviour of $C(t, s)$ is encountered in the dynamical scaling regime $t - s \gg s \gg 1$ where one has the simple scaling form

$$C(t, s) = s^{-b} f_c(t/s). \tag{1}$$

Here f_c is a dynamical scaling function, whereas b is a nonequilibrium exponent. Plotting C as a function of t/s indeed yields an excellent data collapse for $b = 0$, see Figure 2b.

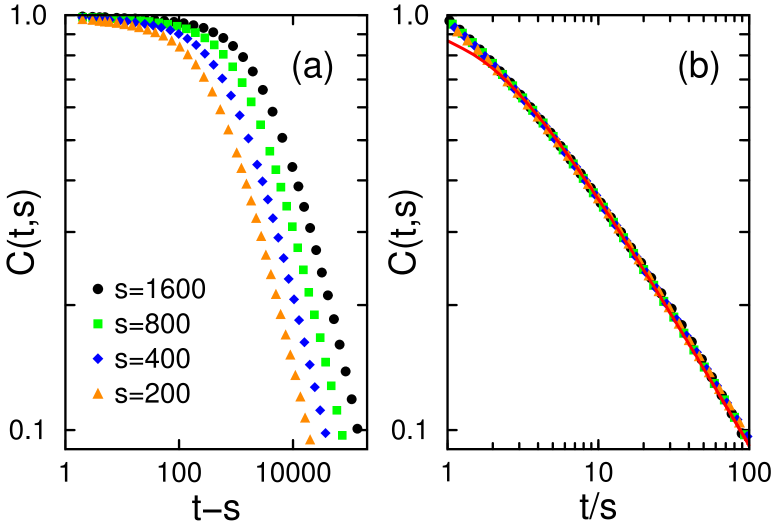

Figure 2. Computed autocorrelation function $C(t,s)$ of the two-dimensional Ising model after a quench to $T = 0$ for different values of s as a function of (a) $t - s$ and (b) t/s. The red line in (b) is the theoretical prediction coming from the theory of local scale invariance.

The red line shown in that Figure is the theoretical prediction derived under the assumption of local scale invariance[5]. Clearly, the theoretical curve nicely describes the numerical data for values of $t/s \geq 2$. The observed deviations for smaller values of t/s are expected as here one is not yet in the dynamical scaling regime $t - s \gg s \gg 1$.

The same conclusions are reached when looking at response functions instead of correlation functions. As an example one may consider the space-time response which gives the reaction of the system at time t at a position $\vec{r}$ to a perturbation which was applied at an earlier time s at a different position $\vec{r}'$. Figure 3 displays the spatially and temporally integrated response which is easily accessible in numerical simulations. Predictions coming from the theory of local scale invariance (full lines) again describe the numerical data in a perfect way[2,4].

It has to be stressed that a similar good agreement between theory and numerics is also found for other systems quenched below their critical point[6]. This leads us to the important conclusion that the generalized space-time symmetries, underlying the theory, are indeed realized in systems that undergo phase ordering.

However, there do exist situations for which local scale invariance does not yield the exact scaling functions. This is for example the case when one quenches a ferromagnet to its critical point. Whereas early simulations showed a very good agreement between theory and numerics also for this case[2], field theoretical calculations[7] pointed to the existence of corrections to the scaling functions derived under the assumption of local scale invariance. We recently succeeded in proving numerically the existence of these correction terms in critical systems by studying the response of the total magnetization to a homogeneous external field[8]. As shown in Figure 4 for the critical Ising model in two and three

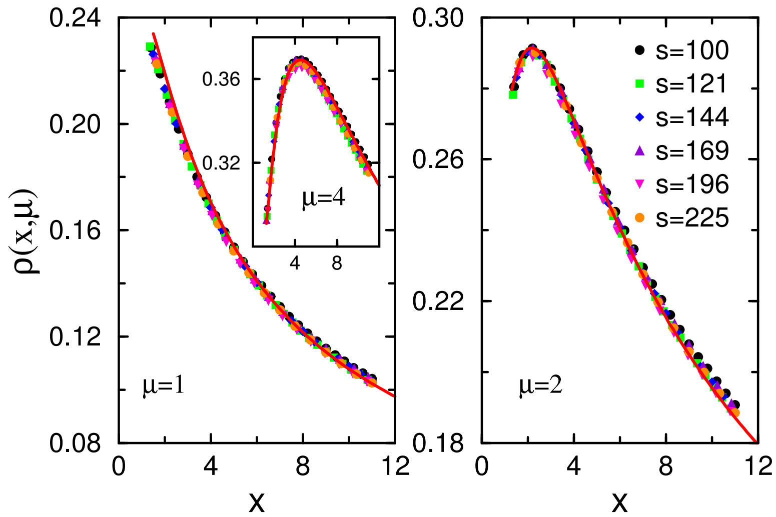

Figure 3. Computed scaling function $\rho(x, \mu)$ of the spatially and temporally integrated response of the two-dimensional Ising model after a quench to $T = 1.5$ for different values of s. Here x is the scaling variable t/s and μ is a measure of the distance over which one has integrated spatially. The red lines are the theoretical predictions coming from the theory of local scale invariance.

dimensions the local scale invariance prediction systematically deviates from the numerical data, whereas the inclusion of the corrections brings the theoretical curve closer to the data. In fact, this is the first time that the existence of corrections to the predictions of the theory of local scale invariance has been proven unambiguously.

Why are quenches to the critical point so different from quenches into the low temperature phase? In fact at a critical point one has no well-defined domains, but instead correlated regions are formed. One can define a dynamical correlation length which increases again with a power law of time, but now the dynamical exponent is different from 2. As a further consequence of the critical fluctuations the time evolution of the magnetization is non-markovian, a feature not captured by the theory of local scale invariance in its present form.

3 Ageing in Critical Spin Glasses

Recently, we have extended our studies to disordered systems. As a first example we looked at the ageing phenomenology of spin glasses at their critical point. Spin glasses are highly frustrated systems which are characterized by very slow dynamics. The nature of their low temperature phase is still intensively debated. In fact, due to the very slow dynamics it is extremely difficult to equilibrate in numerical simulations even systems of very moderate size, making equilibrium simulations of spin glasses very tedious. In our approach we concentrated on the out-of-equilibrium behaviour which can be studied for large systems.

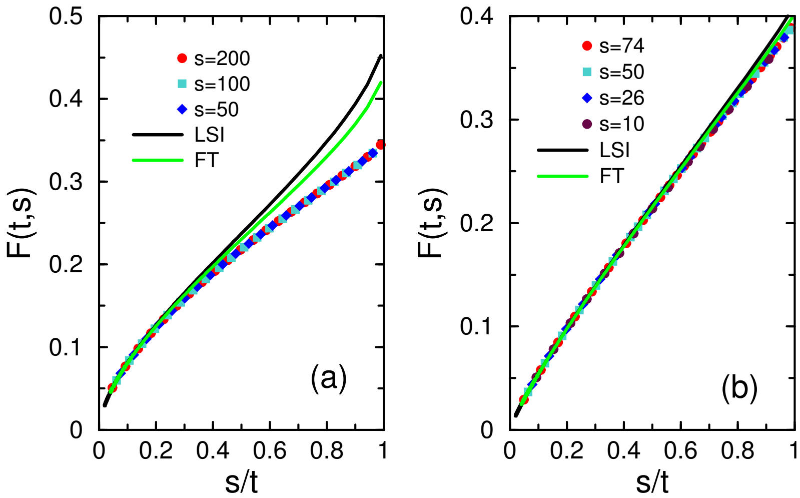

Figure 4. Scaling function of the reponse of the total magnetization to an homogeneous external field for (a) the two-dimensional and (b) the three-dimensional Ising model quenched to its critical point. LSI denotes the theoretical prediction coming from the theory of local scale invariance. In the FT curve the corrections coming from field theoretical calculations have been included.

We focused on the spin glass transition, as earlier investigations had shown that at this point one again has the situation that the dynamical correlation length increases with a power law of time, similar to what is observed in critical ferromagnets[9]. In fact, the similarities with the out-of-equilibrium behaviour of critical ferromagnets go much farther. Figure 5 shows the behaviour of the autocorrelation function after a quench to the critical point for a typical spin glass, the so-called Edwards-Anderson Ising spin glass with a bimodal distribution of the random couplings. Plotted against the time difference $t - s$, ageing is again obvious, see Figure 5a. Assuming the validity of the scaling ansatz (1) also for critical spin glasses, one obtains a perfect scaling behaviour as a function of t/s when choosing the appropriate value of the nonequilibrium exponent b, as shown on Figure 5b. The same observation holds when considering other dynamical two-time quantities. Thus the phenomenology of ageing at the critical point of spin glasses is the same as for critical ferromagnets, and this despite the fact that the free energy surface of spin glasses is very rugged due to frustration effects[10].

Looking a little bit more closely into the out-of-equilibrium behaviour of critical spin glasses, one nevertheless can identify unexpected features which greatly differ from the behaviour of the corresponding ferromagnetic systems. At a critical point it is expected that quantities like critical exponents or critical amplitude ratios are universal and do not depend on details of the considered system (as long as there are no global features involved, like the dimensionality of the system or the symmetry of the order parameter, which lead to a change of the universality class). The origin of this universal behaviour is well understood and has been verified in numerous cases. However, looking at nonequilibrium quantities in critical spin glasses one has the surprise that these quantities, which have been shown to be universal in critical ferromagnets, depend on the choice of the distribution of the

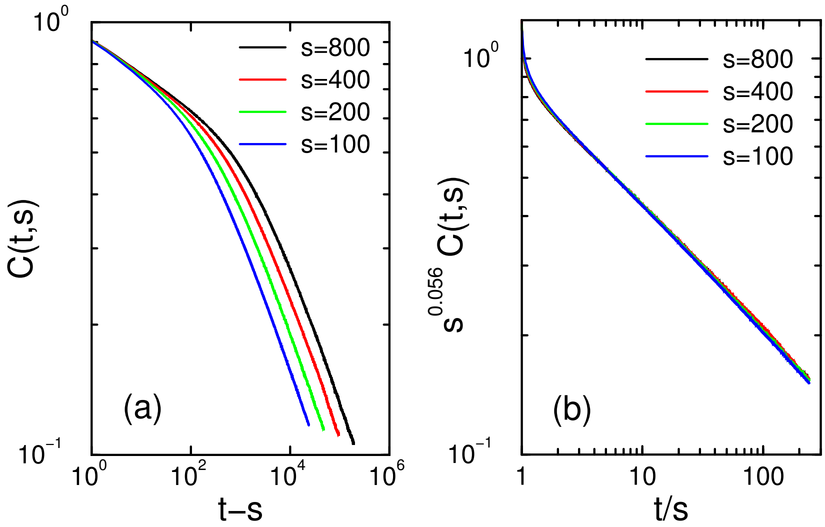

Figure 5. Autocorrelation function $C(t,s)$ as a function of (a) $t-s$ and (b) t/s for the three-dimensional Edwards-Anderson Ising spin glass with a bimodal distribution of the couplings after a quench to the critical point. As shown in (b) the scaling ansatz (1) yields an excellent data collapse.

	bimodal	gaussian	laplacian
b	0.056(3)	0.043(1)	0.032(2)
λ/z	0.362(5)	0.320(5)	0.259(2)
X_∞	0.12(1)	0.09(1)	0.055(2)

Table 1. Various nonequilibrium quantities determined numerically in the critical three-dimensional Edwards-Anderson Ising spin glass for different distributions of the random couplings.

couplings[11]. One example is given by the already mentioned exponent b whose value depends on the distribution function. This can be seen in Table 1 where the values of b are displayed for Edwards-Anderson spin glasses in three space dimensions with three different distributions of the couplings: bimodal, gaussian and laplacian. Also given are two other quantities, again supposed to be universal: the exponent λ/z which governs the decay of the autocorrelation function for large times, $C(t,s=0) \sim t^{-\lambda/z}$, and the limit value X_∞ of the so-called fluctuation-dissipation ratio X which can be used to assign an effective temperature to our nonequilibrium system[1]. It is obvious from the table that these quantities also depend on the choice of the distribution function. This fact is illustrated in Figure 6 where I show $C(t,s=0)$ and $X(s/t)$ for the three studied distributions.

Our nonequilibrium simulations yield therefore evidence that universality in critical spin glasses is much weaker than in critical ferromagnets. The dependence of critical quantities on the choice of the distribution of the couplings is unexpected and right now not well understood. It must be noticed, however, that equilibrium simulations of small systems also indicate a dependence of static quantities on the distribution function[12], even though the situation is not as clear-cut as it is for our nonequilibrium simulations.

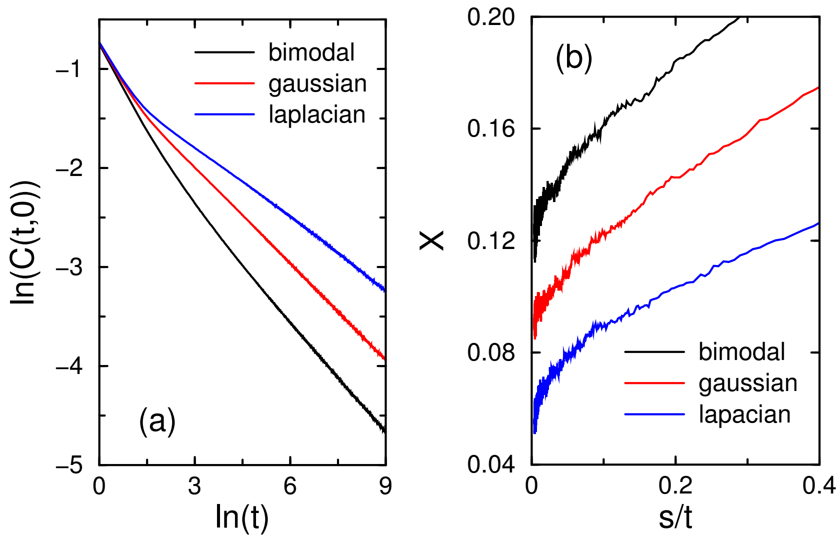

Figure 6. (a) $C(t, s = 0)$ as a function of time t and (b) fluctuation-dissipation ratio X as a function of s/t (yielding the limit value X_∞ when $s/t \longrightarrow 0$) for the three-dimensional Edwards-Anderson Ising spin glass with different distributions of the couplings.

4 Conclusion

In this contribution I have discussed ageing phenomena and dynamical scaling in systems which are brought out of equilibrium by a sudden change of temperature. For systems undergoing phase ordering one finds complete agreement between numerical simulations and the predictions coming from the recently developed theory of local scale invariance. At criticality, the existence of correction terms to the theoretical predictions has been proven numerically. Interestingly, critical spin glasses have the same phenomenology of ageing as critical ferromagnets. Surprisingly, for critical spin glasses the values of the studied nonequilibrium quantities depend on the choice of the distribution function of the random couplings. This points to the possibility that universality in critical spin glasses is weaker than in critical ferromagnets.

Acknowledgments

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